Imposing (Non-)Directionality in Dynamic Network Models

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- First Years!

Purpose of Today's Talk

- Show that identified, directed contemporaneous networks can be achieved through the Unified Structural Equation Model (uSEM) and the Group Iterative Multiple Model Estimation (GIMME) procedure
- Demonstrate that uSEM and GIMME can be transformed to an equivalent network of non-directed contemporaneous partial correlations; that is, a Graphical VAR (gVAR)
- Discuss benefits and shortcomings of uSEM, GIMME, and gVAR

General Overview

- What is 'contemporaneous'?
- Why use arrows?
- Brief review of the standard VAR
- Identified and directed networks w/uSEM and GIMME
- The one-way road to the Graphical VAR
- Provide simulated and empirical examples of network models applied to identical data

The Meaning of 'Contemporaneous' Effects

- Any set of relationships that occur at a timescale *shorter* than the specified lag can be picked up by contemporaneous models
  - E.x., Sprinklers → Wet Grass[1]
  - If we measure at intervals of 8-hours we are unlikely to capture Sprinklers being on in one window and Wet Grass in another
- Put another way, lagged relationships rely on causes being in separate windows of time than their effects
- If mechanism for change happens faster than our observations then we may see the effects appear as contemporaneous effects

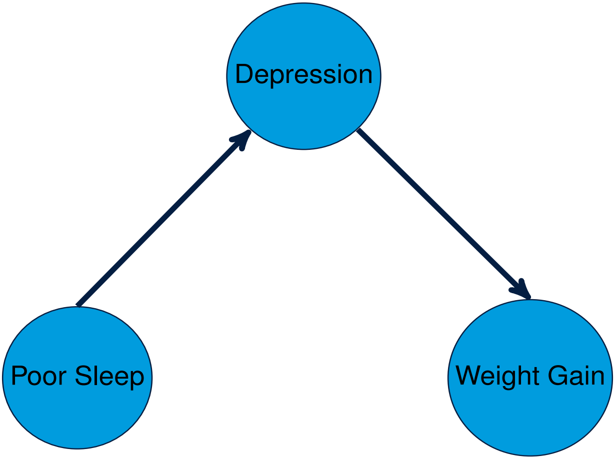
[1] Epskamp et al., 2018

Why Use Arrows?

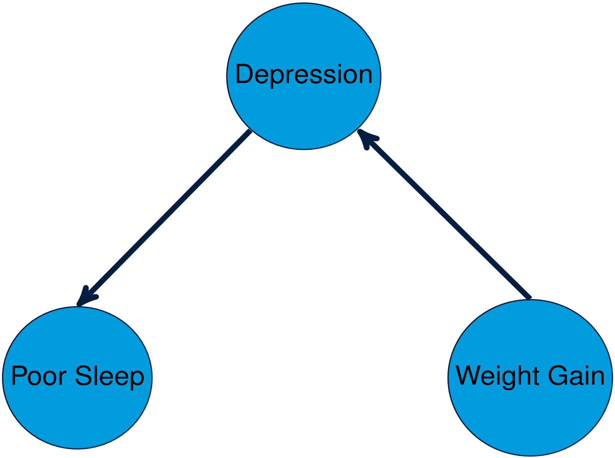
Modeling Directional Relations in Network Models

- At times, we may have theoretical reasons for imposing directional relationships (i.e., causal paths) between variables
  - E.g., Giving QuantDev Talks $\rightarrow$ Fear
- Identifying directional relationships is empirically useful for identifying causal relationships
- In other instances, causal relationships can be difficult to pin down and directionality can be difficult to determine
- Formally, there is an issue with identifiability in directional models

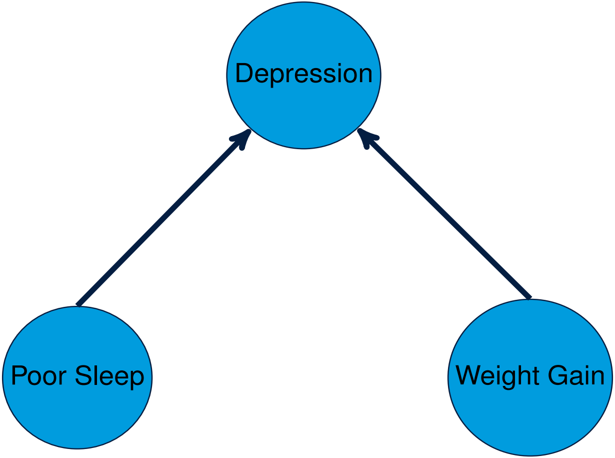
Identifiability In Directed Networks



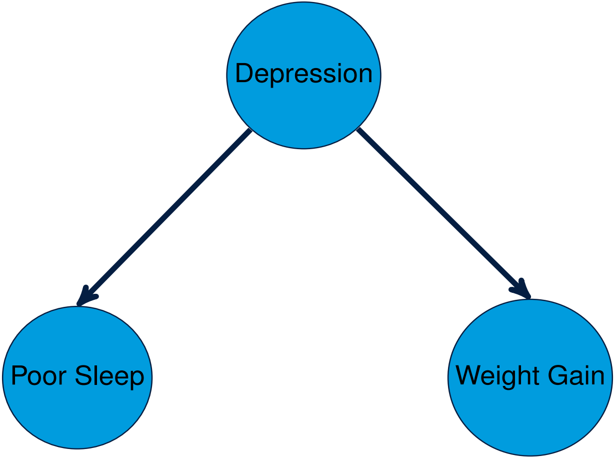
Identifiability In Directed Networks



Identifiability In Directed Networks



Identifiability In Directed Networks

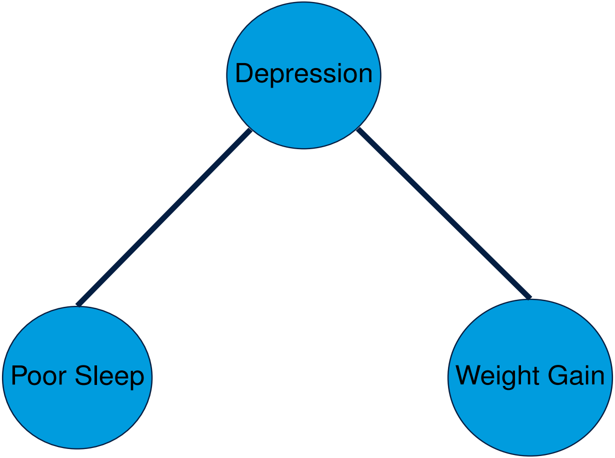


Identifiability In Directed Networks

Conditional Relationships

- Thus, we can only be certain that Weight Gain and Poor Sleep–in this example–are conditionally independent given Depression
- This interpretation yields a single solution and can be represented with non-directional edges that represent partial correlations (i.e., relations between variables after conditioning upon all other variables)

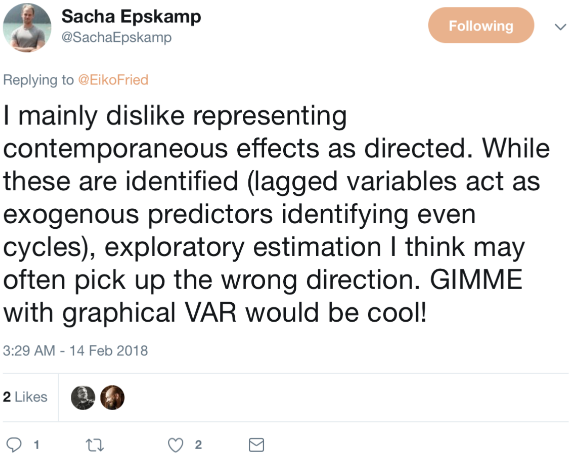
Identifiability In Directed Networks



Recap

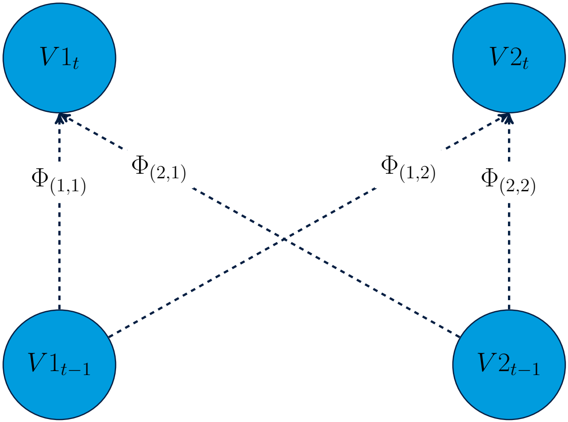
- Contemporaneous networks can improve the power of temporal networks by assessing windows of time between the specified lag
- Directed networks can be difficult to identify as many combinations of directional relationships may describe an observed covariance structure
- At times, systems may have bi-directional relationships where reciprocity is baked into the model thus, imposing uni-directional, causal paths may be inappropriate

Thus, some are skeptical of estimating directed networks



The Standard VAR

A Visual Representation



[2] In this instance, circles represent observed indicators

Standard VAR

$$\eta_{(t)} = c + \mathbf{\Phi}^* \eta_{(t-1)} + \zeta^*_{(t)}$$

where, for a p-variate time-series:

- $\eta_{(t)}$ = p-variate vector of scores at time, t
- c = p-variate vector of constants
- $\mathbf{\Phi}^*$ = $p*p$-dimensional matrix of lagged regression coefficients
- $\eta_{(t-1)}$ = p-variate vector of scores at a given lag
- $\zeta^*_{(t)}$ = p-variate vector of residuals
    - $\mathbf{\Psi}^* = cov[\zeta^*_{(t)}, \zeta^{*t}_{(t)}]$ is a full covariance matrix containing information of contemporaneous relations and noise.

Estimating Contemporaneous Relations

$$\mathbf{\Psi}^* = cov[\zeta^*_{(t)}, \zeta^{*t}_{(t)}]$$

- A path model may be fitted to the full residual covariance matrix from the standard VAR (Kim et al., 2007; Gates et al., 2010)
  - This can be done to extract contemporaneous relations conditioned upon the lagged associations between variables
  - Directed Regression Weights (Structural VAR; sVAR)
- Alternatively, the inverse of the residual covariance matrix (i.e., $\mathbf{K} = \mathbf{\Psi}^{*^{-1}}$) may be used to calculate conditional relations between variables
  - Non-Directed Partial Correlations (Graphical VAR; gVAR)

Structural VAR

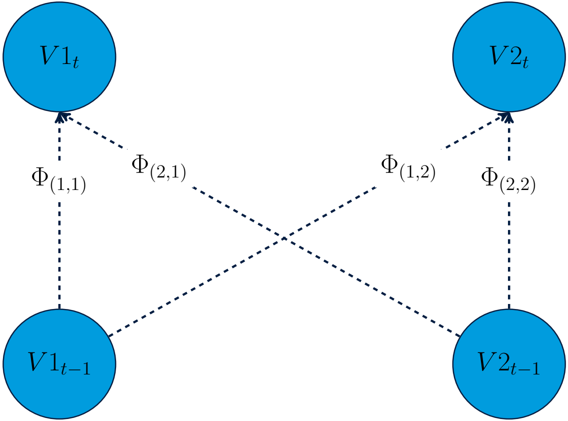
$$\eta_{(t)} = c + \mathbf{A}\eta_{(t)} + \mathbf{\Phi}\eta_{(t-1)} + \zeta_{(t)}$$

where:

- $\eta_{(t)} = p$-variate vector of scores at time, t
- $c = p$-variate vector of constants
- $\mathbf{A} = p*p$-dimensional matrix of contemporaneous regression coefficients with 0's along the diagonal
- $\mathbf{\Phi} = p*p$-dimensional matrix of lagged regression coefficients
- $\eta_{(t-1)} = p$-variate vector of scores at a given lag
- $\zeta_{(t)} = p$-variate vector of residuals
  - $\mathbf{\Psi} = cov[\zeta_{(t)}, \zeta_{(t)}^t]$ is a diagonal covariance matrix

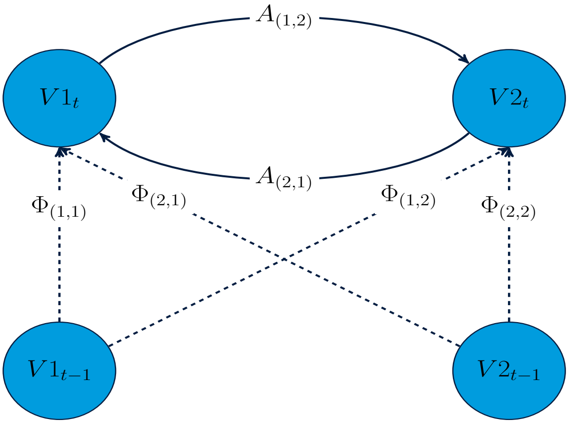
Standard VAR → Structural VAR

A Diagram



Standard VAR → Structural VAR

A Diagram



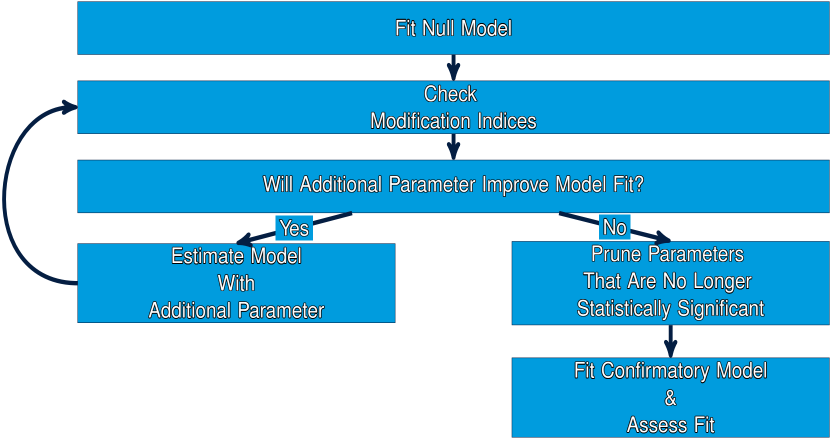
Unified Structural Equation Model (uSEM)

Iterative Approach to the sVAR

- A special case of the Structural VAR
- Simultaneously estimates elements of $\mathbf{\Phi}$ and $\mathbf{A}$ using full information maximum likelihood (FIML)
- Seeks to generate a sparsely parameterized solution via iterative assessments of modification indices
- There is a chance that a symmetry will exist in the modification indices of the contemporaneous network
  - I.e., $X \rightarrow Y$ and $X \leftarrow Y$ improve model fit by the same degree
  - By freeing the autoregressions for estimation first, any symmetry in the contemporaneous elements will be broken and elements of $\mathbf{A}$ will no longer be symmetric

Unified Structural Equation Model (uSEM)

The Iterative Method



Group Iterative Multiple Model Estimation (GIMME) Procedure

$$\eta_{(t)} = c + (\mathbf{A}_i + \mathbf{A}_g)\eta_{(t)} + (\mathbf{\Phi}_i + \mathbf{\Phi}_g)\eta_{(t-1)} + \zeta_{(t)}$$

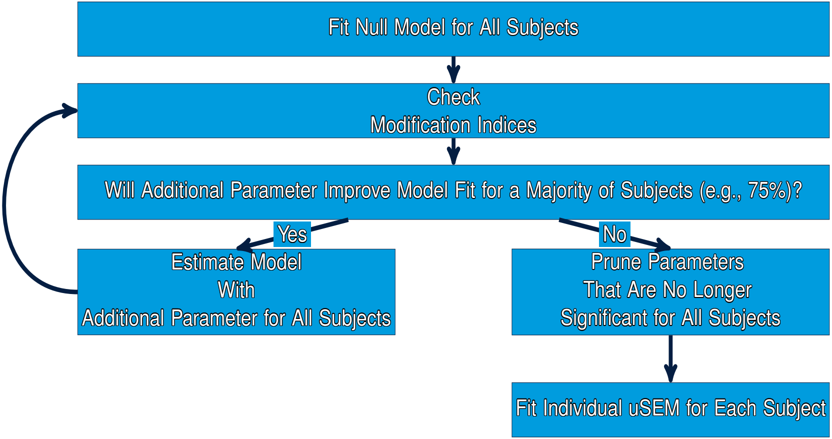
where:

- $\eta_{(t)} = p$-variate vector of scores at time, t
- $c = p$-variate vector of constants
- $\mathbf{A} = p{*}p$-dimensional matrix of contemporaneous regression coefficients with 0's along the diagonal; $(_g)$ indicates group-membership for subject $(_i)$
- $\mathbf{\Phi} = p{*}p$-dimensional matrix of lagged regression coefficients
- $\eta_{(t-1)} = p$-variate vector of scores at a given lag
- $\zeta_{(t)} = p$-variate vector of residuals
  - $\mathbf{\Psi} = cov[\zeta_{(t)}, \zeta^t_{(t)}]$ is a diagonal covariance matrix

[3] Equations from Beltz & Gates, 2017

Group Iterative Multiple Model Estimation Procedure

GIMME



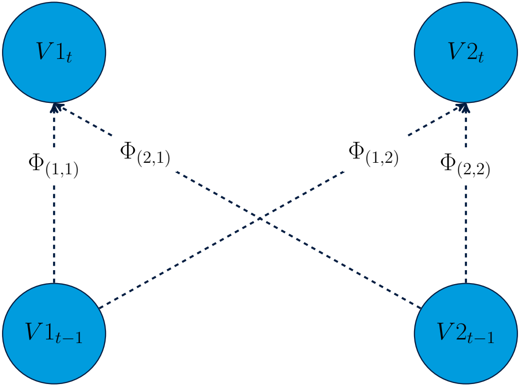
Recap

Structural VAR, uSEM, and GIMME

- The structural VAR model adds in a network of contemporaneous regression weights in addition to the standard VAR model's lagged regressions
- The addition of contemporaneous effects helps capture effects within the lag as well as removes spurious relations from the autoregressions
- uSEM and GIMME are special cases of the structural VAR and can work around the issue of identifiability by freeing autoregressive terms for estimation prior to freeing the contemporaneous effects

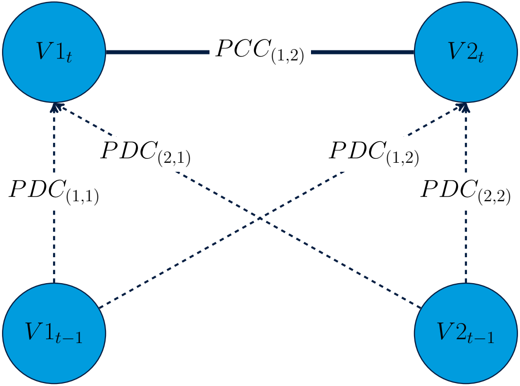
Standard VAR → Graphical VAR

A Diagram



Standard VAR → Graphical VAR

A Diagram



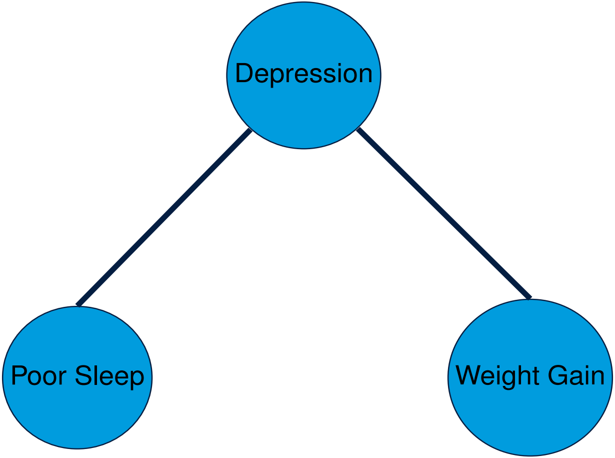
[4]Change in notation is due to standardization of elements of $\mathbf{\Phi}^*$

Non-Directional Networks

- At times, it may not make theoretical sense to impose a directional network of relationships
  - For instance, psychopathology[5]
- In these instances, it may be useful to have a non-directional network
  - The trade off for certainty in model identification is directionality

[5] Borsboom & Cramer, 2013

A Non-Directional Network Example



Graphical VAR

Conceptualization

- Graphical VAR generates a network of contemporaneous partial correlations using the inverse of the residual covariance matrix from the standard VAR
  - i.e., $\mathbf{K} = \mathbf{\Psi}^{*-1} = cov[\zeta^*_{(t)}, {\zeta^*_{(t)}}^t]^{-1}$
- Makes use of LASSO regularization to penalize non-zero partial correlation estimates that may be due to sampling variation
- Results can be interpreted as X $\leftrightarrow$ Y conditioned upon all other pairwise associations
- Also generates a network of standardized, lagged regression coefficients

VAR $\rightarrow$ gVAR - The Partial Contemporaneous Network

The inverse of the residual covariance matrix (i.e., $\mathbf{K} = \mathbf{\Psi}^{*-1}$) can be transformed into a network of partial contemporaneous correlations (PCCs) using the following:

$$PCC(X_{i(t)}, X_{j(t)}) = -\frac{K_{ij}}{\sqrt{K_{ii}K_{jj}}}$$

Where:

- K_{ij} is the element of $\mathbf{K}$ at coordinates (i, j)
- K_{ii} is the diagonal element of $\mathbf{K}$ associated with item, i
- K_{jj} is the diagonal element of $\mathbf{K}$ associated with item, j

VAR → gVAR - The Partial Directed Network

The lagged relationships of the VAR can also be standardized using information from **K** to form a network of partial directed correlations (PDCs) using the following formula:

$$PDC(X_{i(t)}, X_{j(t-1)}) = \frac{\mathbf{\Phi}^*_{ij}}{\sqrt{\mathbf{\Psi}^*_{ii} K_{jj} + \mathbf{\Phi}^{*2}_{ij}}}$$

Where:

- $\mathbf{\Phi}^*_{ij}$ is the regression coefficient of i on j
- $\mathbf{\Psi}^*_{ii}$ is the residual variance of the outcome at time, t
- K_{jj} is the diagonal element of **K** associated with item, j

Recap

- The graphical VAR trades directionality for certainty in model identification
- This is achieved by inverting the residual covariance matrix of the standard VAR
- Regression weights from the standard VAR (i.e., $\mathbf{\Phi}^*$) are standardized in addition
- LASSO regularization penalizes non-zero elements allowing for sparsely parameterized models

Structural VAR → Standard VAR

- Any sVAR can be backtransformed into a standard VAR by moving $\eta_{(t)}$ to one side of the equation

$$\eta_{(t)} = \mathbf{A}\eta_{(t)} + \mathbf{\Phi}\eta_{(t-1)} + \zeta_{(t)} \rightarrow$$

$$\eta_{(t)} = (I_p - \mathbf{A})^{-1}\mathbf{\Phi}\eta_{(t-1)} + (I_p - \mathbf{A})^{-1}\zeta_{(t)}$$

Structural VAR → Standard VAR

$$\eta_{(t)} = (I_p - \mathbf{A})^{-1}\mathbf{\Phi}\eta_{(t-1)} + (I_p - \mathbf{A})^{-1}\zeta_{(t)}$$

where:

- $\mathbf{\Phi}^* = (I_p - \mathbf{A})^{-1}\mathbf{\Phi}$
- $\zeta^*_{(t)} = (I_p - \mathbf{A})^{-1}\zeta_{(t)}$

We obtain:

$$\eta_{(t)} = \mathbf{\Phi}^*\eta_{(t-1)} + \zeta^*_{(t)}$$

and $\mathbf{\Psi}$ is transformed into:

$$\mathbf{\Psi}^* = (I_p - \mathbf{A})^{-1}\mathbf{\Psi}(I_p - \mathbf{A}^t)^{-1}$$

Which is the Standard VAR

Recap

- Relatively simple transformation can revert a structural VAR (e.g., uSEM and GIMME) into a standard VAR
- Once transformed into a standard VAR, the graphical VAR equations can be applied to the newly obtained $\mathbf{\Phi}^*$ and $\mathbf{\Psi}^*$

Comment on sVAR $\rightarrow$ gVAR

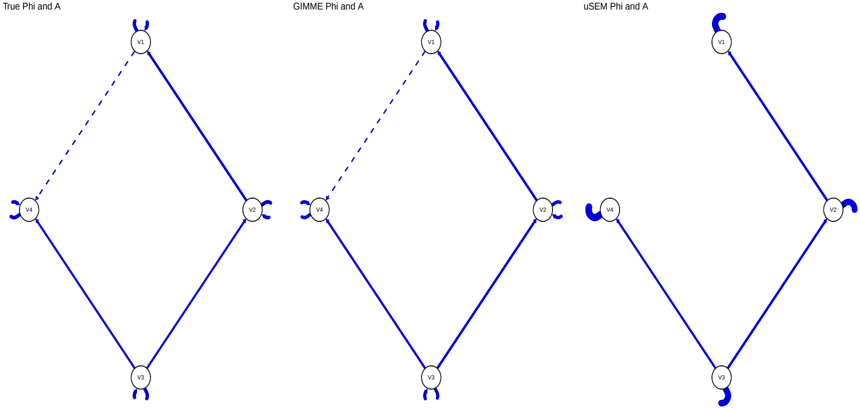
- In moving from structural VAR to graphical VAR we keep information as to the direction of contemporaneous effects
- If starting from a standard VAR or graphical VAR, $\mathbf{\Phi}^*$ and $\mathbf{\Psi}^*$ contain information of contemporaneous effects that cannot be parsed apart without fitting a new structural VAR
- Thus, the transformation is unidirectional

Simulation Example

- Simulate a structural VAR (e.g., $\mathbf{A}$, $\mathbf{\Phi}$, $\mathbf{\Psi}$)
- Compare parameter recovery of uSEM and GIMME against true structural VAR
- Transform true model, uSEM, and GIMME to graphical VARs and compare against graphical VAR on equivalent metric

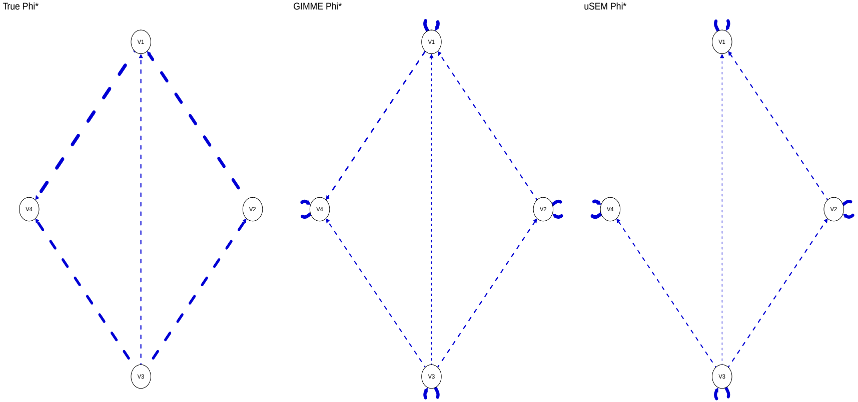
Simulation Example

Structual VAR



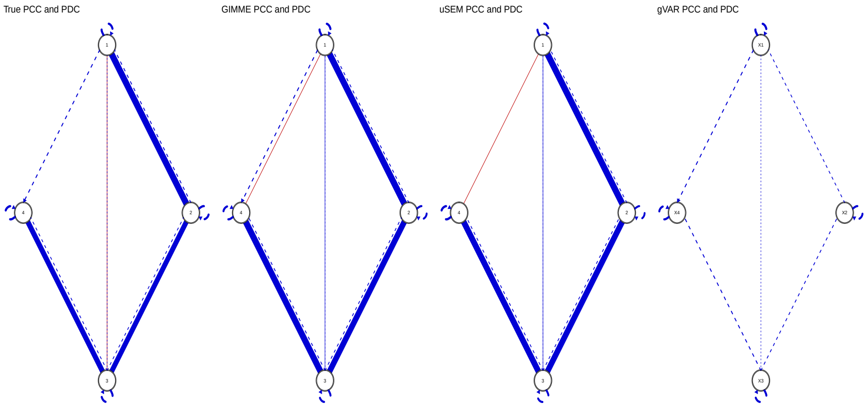
Simulation Example

Standard VAR



Simulation Example

Graphical VAR



Simulation

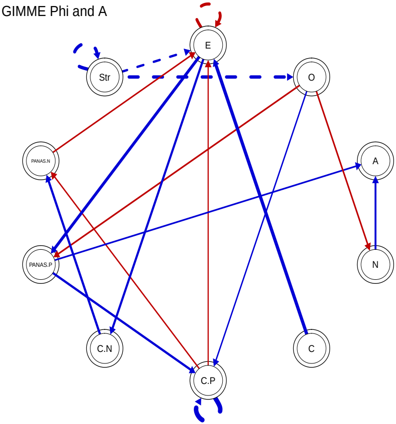
To recap:

- In structural VAR comparison, uSEM failed to capture lagged association between $V1$ and $V4$
- In standard VAR, phantom edges appear showing the need to correctly specify a structural VAR if the data-generating model is indeed one
- uSEM fails to capture lagged association between $V1$ and $V4$
- The graphical VAR seems to penalize most contemporaneous relations down to low-enough values such that they are no longer statistically significant upon bootstrapping

Empirical Example

Data

- $N = 32$ subjects; $N_t \geq 65$
  - 4-measurements per day, unequally spaced intervals
  - Aggregated into 2-measurements per day (i.e., morning-afternoon and evening-night)
- 10-nodes comprised of:
  - 5-factors from NEO
  - $\pm$ PANAS
  - $\pm$ Circumplexity Inventory
  - Perceived Stress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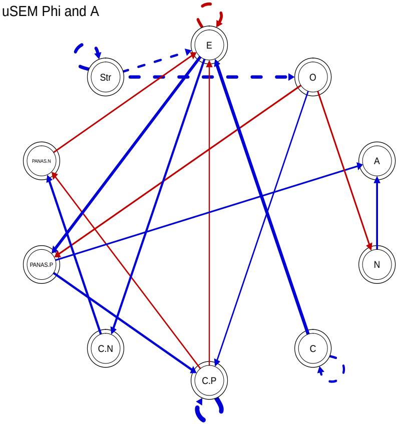
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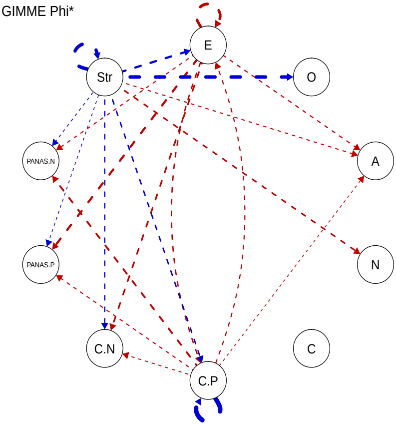
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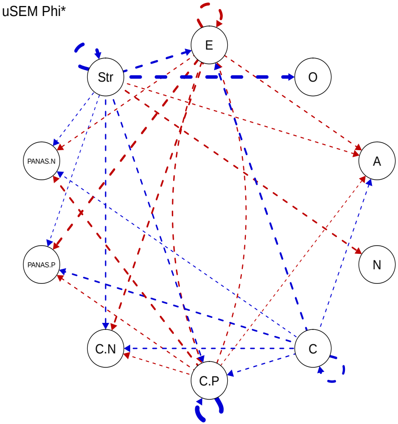
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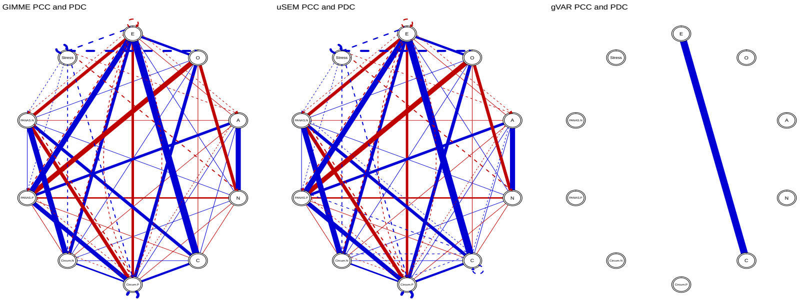
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Current Issues

- Standard errors (SEs) of uSEM and GIMME are difficult to transform over to the metric of the graphical VAR for outright comparisons to be made
  - Graphical VAR provides bootstrapped SEs via *bootnet*-package in R
- Current simulation study assumes that data-generating structure is a structural VAR
- γ-tuning parameter of the LASSO needs to be manipulated but optimal setting is difficult to identify

Thank you!

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